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Abstract
Under the hypotheses of analyticity in the coupling constant, locality, Lorentz covariance and Poincaré invariance of the deformations, combined with the preservation of the number of derivatives on each field, the consistent interactions between a collection of free massless tensor gauge fields with the mixed symmetry of a two-column Young diagram of the type (3,1) and one Abelian vector field, respectively a p-form gauge field, are addressed. The main result is that a single mixed-symmetry tensor field from the collection gets coupled to the vector field/p-form. Our final result resembles the well-known fact from general relativity according to which there is one graviton in a given world.

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1. Introduction

Tensor fields in 'exotic' representations of the Lorentz group, characterized by a mixed Young symmetry type [1–7], held the attention lately on some important issues, such as the dual formulation of field theories of spin two or higher [8–14], the impossibility of consistent cross-interactions in the dual formulation of linearized gravity [15], a Lagrangian first-order approach [16, 17] to some classes of massless or partially massive mixed-symmetry type tensor gauge fields, suggestively resembling to the tetrad formalism of general relativity or the derivation of some exotic gravitational interactions [18, 19]. An important matter related to mixed-symmetry type tensor fields is the study of their consistent interactions, among themselves as well as with higher-spin gauge theories [20–28]. The most efficient approach to this problem is the cohomological one, based on the deformation of the solution to the master equation [29]. The purpose of this paper is to investigate the consistent interactions between a collection of massless tensor gauge fields, each with the mixed symmetry of a two-column

Young diagram of the type (3, 1) and one vector field, respectively one *p*-form gauge field. It is worth mentioning the duality of a free massless tensor gauge field with the mixed symmetry (3, 1) to the Pauli–Fierz theory in $D = 6$ dimensions and, in this respect, some developments concerning the dual formulations of linearized gravity from the perspective of *M*-theory [30–32]. Our analysis relies on the deformation of the solution to the master equation by means of cohomological techniques with the help of the local BRST cohomology, whose component in a single (3, 1) sector has been reported in detail in [33]. This paper generalizes our results from [34] regarding the cross-interactions between a single massless (3, 1) field and a vector field. Under the hypotheses of analyticity in the coupling constant, locality, Lorentz covariance and Poincaré invariance of the deformations, combined with the preservation of the number of derivatives on each field, we find a deformation of the solution to the master equation that provides nontrivial cross-couplings. This case corresponds to a $(p+4)$-dimensional spacetime and is described by a deformed solution that stops at order 2 in the coupling constant. The interacting Lagrangian action contains only mixing-component terms of order 1 and 2 in the coupling constant, *but only one mixed-symmetry tensor field from the collection gets coupled to the p-form, while the others remain free.* At the level of the gauge transformations, only those of the *p*-form are modified at order 1 in the coupling constant with a term linear in the antisymmetrized first-order derivatives of a single gauge parameter from the (3, 1) sector such that the gauge algebra and the reducibility structure of the coupled model are not modified during the deformation procedure, being the same as in the case of the starting free action. Our result is interesting since it *exhibits strong similarities to the Einstein gravitons from general relativity, in the sense that no nontrivial cross-couplings between different fields with the mixed symmetry* (3, 1) *are allowed, neither direct nor intermediated by a p-form.*

2. Free model for *p* = 1: BRST symmetry

We begin with the Lagrangian action

$$
S_0\big[t^A_{\lambda\mu\nu|\alpha}, V_\mu\big] = \int \mathrm{d}^D x \left\{ \frac{1}{2}\big[\big(\partial^\rho t_A^{\lambda\mu\nu|\alpha}\big)\big(\partial_\rho t^A_{\lambda\mu\nu|\alpha}\big) - \big(\partial_\alpha t_A^{\lambda\mu\nu|\alpha}\big)\big(\partial^\beta t^A_{\lambda\mu\nu|\beta}\big)\big] \right.
$$
$$
- \frac{3}{2}\big[\big(\partial_\lambda t_A^{\lambda\mu\nu|\alpha}\big)\big(\partial^\rho t^A_{\rho\mu\nu|\alpha}\big) + \big(\partial^\rho t_A^{\lambda\mu}\big)\big(\partial_\rho t^A_{\lambda\mu}\big)\big] + 3\big(\partial_\alpha t_A^{\lambda\mu\nu|\alpha}\big)\big(\partial_\lambda t^A_{\mu\nu}\big)
$$
$$
\left. + 3\big(\partial_\rho t_A^{\rho\mu}\big)\big(\partial^\lambda t^A_{\lambda\mu}\big) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right\} \equiv S_0^{\mathrm{t}}\big[t^A_{\lambda\mu\nu|\alpha}\big] + S_0^{\mathrm{V}}[V_\mu],
\tag{2.1}
$$

in $D \geqslant 5$ spacetime dimensions, with $A = \overline{1, n}$ and $n > 1$. Each massless tensor field $t^A_{\lambda\mu\nu|\alpha}$ has the mixed symmetry (3, 1) and hence transforms according to an irreducible representation of $GL(D, \mathrm{R})$ corresponding to a four-cell Young diagram with two columns and three rows. It is thus completely antisymmetric in its first three indices and satisfies the identity $t^A_{[\lambda\mu\nu|\alpha]} \equiv 0$. The collection indices A, B, etc, are raised and lowered with a quadratic form k_{AB} that defines a positively-defined metric in the internal space. It can always be normalized to δ_{AB} by a simple linear field redefinition, so one can take $k_{AB} = \delta_{AB}$ and re-write (2.1) as

$$
S_0\big[t^A_{\lambda\mu\nu|\alpha}, V_\mu\big] = \int \mathrm{d}^D x \left[\sum_{A=1}^n \mathcal{L}_0^{\mathrm{t}}\big(t^A_{\lambda\mu\nu|\alpha}, \partial_\rho t^A_{\lambda\mu\nu|\alpha}\big) + \mathcal{L}_0^{\mathrm{V}}\big(V_\mu, \partial_\nu V_\mu\big) \right],
\tag{2.2}
$$

where $\mathcal{L}_0^{\mathrm{t}}\big(t^A_{\lambda\mu\nu|\alpha}, \partial_\rho t^A_{\lambda\mu\nu|\alpha}\big)$ is the Lagrangian density for the field A. The field strength of the vector field V_μ is defined in the standard manner by

$$
F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu \equiv \partial_{[\mu} V_{\nu]}.
\tag{2.3}
$$

Everywhere in this paper, it is understood that the notation $[\lambda \cdots \alpha]$ signifies a complete antisymmetry with respect to the (Lorentz) indices between brackets, with the conventions that the minimum number of terms is always used and the result is never divided by the number of terms. The trace of $t^A_{\lambda\mu\nu|\alpha}$ is defined by $t^A_{\lambda\mu} = \sigma^{\nu\alpha} t^A_{\lambda\mu\nu|\alpha}$ and is obviously an antisymmetric tensor. Everywhere in this paper, we employ the flat Minkowski metric of 'mostly plus' signature $\sigma^{\mu\nu} = \sigma_{\mu\nu} = (-, +, +, +, +, \cdots)$.

A generating set of gauge transformations for action (2.1) can be taken of the form

$$\delta_{\epsilon,\chi} t^A_{\lambda\mu\nu|\alpha} = -3\partial_{[\lambda} \epsilon^A_{\mu\nu\alpha]} + 4\partial_{[\lambda} \epsilon^A_{\mu\nu]\alpha} + \partial_{[\lambda} \chi^A_{\mu\nu]|\alpha}, \tag{2.4}$$

$$\delta_\epsilon V_\mu = \partial_\mu \epsilon, \tag{2.5}$$

where the gauge parameters $\epsilon^A_{\lambda\mu\nu}$ determine n completely antisymmetric tensors, the other set of gauge parameters displays the mixed symmetry (2, 1), such that each of them is antisymmetric in the first two indices and satisfies the identity $\chi^A_{[\mu\nu|\alpha]} \equiv 0$, and the gauge parameter ϵ is a scalar. The generating set of gauge transformations (2.4) and (2.5) is off-shell, second-order reducible, the accompanying gauge algebra being obviously Abelian (for details, see [33]).

The construction of the antifield-BRST symmetry for this free theory debuts with the identification of the algebra on which the BRST differential s acts. The generators of the BRST algebra are of two kinds: fields/ghosts and antifields. The ghost spectrum for the model under study comprises the fermionic ghosts $\{\eta^A_{\lambda\mu\nu}, \mathcal{G}^A_{\mu\nu|\alpha}, \eta\}$ associated with the gauge parameters $\{\epsilon^A_{\lambda\mu\nu}, \chi^A_{\mu\nu|\alpha}, \epsilon\}$ from (2.4) and (2.5), the bosonic ghosts for ghosts $\{C^A_{\mu\nu}, \mathcal{C}^A_{\nu\alpha}\}$ due to the first-stage reducibility relations and also the fermionic ghosts for ghosts C^A_ν corresponding to the second-stage reducibility relations. We ask that $\eta^A_{\lambda\mu\nu}$ and $C^A_{\mu\nu}$ are completely antisymmetric, $\mathcal{G}^A_{\mu\nu|\alpha}$ display the mixed symmetry (2, 1) and $\mathcal{C}^A_{\nu\alpha}$ are symmetric. The antifield spectrum is organized into the antifields $\{t^{*\lambda\mu\nu|\alpha}_A, V^{*\mu}\}$ of the original tensor fields, together with those of the ghosts, $\{\eta^{*\lambda\mu\nu}_A, \mathcal{G}^{*\mu\nu|\alpha}_A, \eta^*\}$, $\{C^{*\mu\nu}_A, \mathcal{C}^{*\nu\alpha}_A\}$ and respectively $C^{*\nu}_A$, of statistics opposite to that of the associated fields/ghosts. It is understood that $t^{*\lambda\mu\nu|\alpha}_A$ exhibit the same mixed-symmetry properties like $t^A_{\lambda\mu\nu|\alpha}$ and similarly with respect to $\eta^{*\lambda\mu\nu}_A, \mathcal{G}^{*\mu\nu|\alpha}_A, C^{*\mu\nu}_A$ and $\mathcal{C}^{*\nu\alpha}_A$. For subsequent purpose, we denote the trace of $t^{*\lambda\mu\nu|\alpha}_A$ by $t^{*\lambda\mu}_A$, being understood that it is antisymmetric.

Since both the gauge generators and reducibility functions for this model are field-independent, it follows that the BRST differential s simply reduces to

$$s = \delta + \gamma, \tag{2.6}$$

where δ represents the Koszul–Tate differential, graded by the antighost number agh $(\mathrm{agh}(\delta) = -1)$ and γ stands for the exterior derivative along the gauge orbits, whose degree is named the pure ghost number pgh $(\mathrm{pgh}(\gamma) = 1)$. The overall degree that grades the BRST complex is known as the ghost number (gh) and is defined as the difference between the pure ghost number and the antighost number, such that $\mathrm{gh}(s) = \mathrm{gh}(\delta) = \mathrm{gh}(\gamma) = 1$. According to the standard rules of the BRST method, the corresponding degrees of the generators from the BRST complex are valued as

$$\mathrm{pgh}\left(\eta^A_{\lambda\mu\nu}\right) = \mathrm{pgh}\left(\mathcal{G}^A_{\mu\nu|\alpha}\right) = \mathrm{pgh}(\eta) = 1,$$
$$\mathrm{pgh}\left(C^A_{\mu\nu}\right) = 2 = \mathrm{pgh}\left(\mathcal{C}^A_{\nu\alpha}\right), \qquad \mathrm{pgh}\left(C^A_\nu\right) = 3,$$
$$\mathrm{agh}\left(t^{*\lambda\mu\nu|\alpha}_A\right) = 1 = \mathrm{agh}(V^{*\mu}),$$
$$\mathrm{agh}\left(\eta^{*\lambda\mu\nu}_A\right) = \mathrm{agh}\left(\mathcal{G}^{*\mu\nu|\alpha}_A\right) = \mathrm{agh}(\eta^*) = 2,$$
$$\mathrm{agh}\left(C^{*\mu\nu}_A\right) = 3 = \mathrm{agh}\left(\mathcal{C}^{*\nu\alpha}_A\right), \qquad \mathrm{agh}\left(C^{*\nu}_A\right) = 4,$$

plus the usual rules that the degrees of the original fields, the antighost number of the ghosts and the pure ghost number of the antifields all vanish. The actions of δ and γ on the generators from the BRST complex are given by

$$\gamma t^A_{\lambda\mu\nu|\alpha} = -3\partial_{[\lambda}\eta^A_{\mu\nu\alpha]} + 4\partial_{[\lambda}\eta^A_{\mu\nu]\alpha} + \partial_{[\lambda}\mathcal{G}^A_{\mu\nu]|\alpha}, \qquad \gamma V_\mu = \partial_\mu\eta, \tag{2.7}$$

$$\gamma\eta^A_{\lambda\mu\nu} = -\tfrac{1}{2}\partial_{[\lambda}C^A_{\mu\nu]}, \qquad \gamma\eta = 0, \tag{2.8}$$

$$\gamma\mathcal{G}^A_{\mu\nu|\alpha} = 2\partial_{[\mu}C^A_{\nu\alpha]} - 3\partial_{[\mu}C^A_{\nu]\alpha} + \partial_{[\mu}\mathcal{C}^A_{\nu]\alpha}, \tag{2.9}$$

$$\gamma C^A_{\mu\nu} = \partial_{[\mu}C^A_{\nu]}, \qquad \gamma\mathcal{C}^A_{\nu\alpha} = -3\partial_{(\nu}C^A_{\alpha)}, \qquad \gamma C^A_\nu = 0, \tag{2.10}$$

$$\gamma t^{*\lambda\mu\nu|\alpha}_A = \gamma V^{*\mu} = \gamma\eta^{*\lambda\mu\nu}_A = \gamma\mathcal{G}^{*\mu\nu|\alpha}_A = \gamma\eta^* = 0, \tag{2.11}$$

$$\gamma C^{*\mu\nu}_A = \gamma\mathcal{C}^{*\nu\alpha}_A = \gamma C^{*\nu}_A = 0, \tag{2.12}$$

$$\delta t^A_{\lambda\mu\nu|\alpha} = \delta V_\mu = \delta\eta^A_{\lambda\mu\nu} = \delta\mathcal{G}^A_{\mu\nu|\alpha} = \delta\eta = 0, \tag{2.13}$$

$$\delta C^A_{\mu\nu} = \delta\mathcal{C}^A_{\nu\alpha} = \delta C^A_\nu = 0, \tag{2.14}$$

$$\delta t^{*\lambda\mu\nu|\alpha}_A = T^{\lambda\mu\nu|\alpha}_A, \qquad \delta V^{*\mu} = -\partial_\nu F^{\nu\mu}, \qquad \delta\eta^{*\lambda\mu\nu}_A = -4\partial_\alpha t^{*\lambda\mu\nu|\alpha}_A, \tag{2.15}$$

$$\delta\mathcal{G}^{*\mu\nu|\alpha}_A = -\partial_\lambda\big(3t^{*\lambda\mu\nu|\alpha}_A - t^{*\mu\nu\alpha|\lambda}_A\big), \qquad \delta\eta^* = -\partial_\mu V^{*\mu}, \tag{2.16}$$

$$\delta C^{*\mu\nu}_A = 3\partial_\lambda\big(\mathcal{G}^{*\mu\nu|\lambda}_A - \tfrac{1}{2}\eta^{*\lambda\mu\nu}_A\big), \qquad \delta\mathcal{C}^{*\nu\alpha}_A = \partial_\mu\mathcal{G}^{*\mu(\nu|\alpha)}_A, \tag{2.17}$$

$$\delta C^{*\nu}_A = 6\partial_\mu\big(\mathcal{C}^{*\mu\nu}_A - \tfrac{1}{3}C^{*\mu\nu}_A\big), \tag{2.18}$$

where $T^{\lambda\mu\nu|\alpha}_A$ are minus the Euler–Lagrange derivatives of action (2.1) with respect to the field $t^A_{\lambda\mu\nu|\alpha}$.

The Lagrangian BRST differential admits a canonical action in a structure named antibracket and defined by decreeing the fields/ghosts conjugated with the corresponding antifields, $s\cdot = (\cdot, S)$, where (,) signifies the antibracket and S denotes the canonical generator of the BRST symmetry. It is a bosonic functional of the ghost number zero (involving both field/ghost and antifield spectra) that obeys the master equation $(S, S) = 0$. The master equation is equivalent to the second-order nilpotency of s, where its solution S encodes the entire gauge structure of the associated theory. Taking into account formulae (2.7)–(2.18) as well as the standard actions of δ and γ in canonical form, we find that the complete solution to the master equation for the free model under study is given by

$$\begin{aligned}
S = S_0\big[t^A_{\lambda\mu\nu|\alpha}, V_\mu\big] + \int \mathrm{d}^D x &\bigg[t^{*\lambda\mu\nu|\alpha}_A\big(3\partial_\alpha\eta^A_{\lambda\mu\nu} + \partial_{[\lambda}\eta^A_{\mu\nu]\alpha} + \partial_{[\lambda}\mathcal{G}^A_{\mu\nu]|\alpha}\big) \\
&- \frac{1}{2}\eta^{*\lambda\mu\nu}_A\partial_{[\lambda}C^A_{\mu\nu]} + \mathcal{G}^{*\mu\nu|\alpha}_A\big(2\partial_\alpha C^A_{\mu\nu} - \partial_{[\mu}C^A_{\nu]\alpha} + \partial_{[\mu}\mathcal{C}^A_{\nu]\alpha}\big) \\
&+ C^{*\mu\nu}_A\partial_{[\mu}C^A_{\nu]} - 3\mathcal{C}^{*\nu\alpha}_A\partial_{(\nu}C^A_{\alpha)} + V^{*\mu}\partial_\mu\eta\bigg] \equiv S^t + S^V.
\end{aligned} \tag{2.19}$$

3. Brief review of the deformation procedure

There are three main types of consistent interactions that can be added to a given gauge theory. The first type deforms only the Lagrangian action, but not its gauge transformations. The second kind modifies both the action and its transformations, but not the gauge algebra. The third, and certainly the most interesting category, changes everything, namely, the action, its gauge symmetries and the accompanying algebra.

The reformulation of the problem of consistent deformations of a given action and of its gauge symmetries in the antifield-BRST setting is based on the observation that if a deformation of the classical theory can be consistently constructed, then the solution to the master equation for the initial theory can be deformed into the solution of the master equation for the interacting theory

$$\bar{S} = S + gS_1 + g^2 S_2 + O(g^3), \qquad \varepsilon(\bar{S}) = 0, \qquad \mathrm{gh}(\bar{S}) = 0, \tag{3.1}$$

such that

$$(\bar{S}, \bar{S}) = 0. \tag{3.2}$$

Here and in the following, $\varepsilon(F)$ denotes the Grassmann parity of F. The projection of (3.1) on the various powers of the coupling constant induces the following tower of equations:

$$g^0 : (S, S) = 0, \tag{3.3}$$

$$g^1 : (S_1, S) = 0, \tag{3.4}$$

$$g^2 : \tfrac{1}{2}(S_1, S_1) + (S_2, S) = 0, \tag{3.5}$$

$$\vdots$$

The first equation is satisfied by hypothesis. The second governs the first-order deformation of the solution to the master equation, S_1 and shows that S_1 is a BRST co-cycle, $sS_1 = 0$. This means that S_1 pertains to the ghost number zero cohomological space of s, $H^0(s)$, which is generically non-empty because it is isomorphic to the space of physical observables of the free theory. The remaining equations are responsible for the higher-order deformations of the solution to the master equation. No obstructions arise in finding solutions to them as long as no further restrictions, such as spacetime locality or Lorentz covariance, are imposed. Obviously, only nontrivial first-order deformations should be considered, since trivial ones ($S_1 = sB$) lead to the trivial deformations of the initial theory and can be eliminated by convenient redefinitions of the fields. Ignoring the trivial deformations, it follows that S_1 is a nontrivial BRST-observable, $S_1 \in H^0(s)$. Once that the deformation equations (3.4) and (3.5), etc, have been solved by means of specific cohomological techniques, from the consistent nontrivial deformed solution to the master equation, one can extract all information on the gauge structure of the resulting interacting theory.

4. Main results for $p = 1$

The aim of this paper is to investigate the consistent interactions that can be added to action (2.1) without modifying either the field spectrum or the number of independent gauge symmetries. This matter is addressed in the context of the antifield-BRST deformation procedure described above and relies on computing the solutions to equations (3.4) and (3.5), etc, from the cohomology of the BRST differential. For obvious reasons, we consider only analytic, local and manifestly covariant deformations and, meanwhile, restrict to the Poincaré-invariant quantities, i.e. we do not allow explicit dependence on the spacetime coordinates. The analyticity of deformations refers to the fact that the deformed solution to the master equation (3.1) can be expanded in a formal power series in the coupling constant g that makes sense and reduces to the original solution (2.19) in the free limit ($g = 0$). Moreover, we ask that the deformed gauge theory preserves the Cauchy order of the uncoupled model, which enforces the requirement that the interacting Lagrangian is of maximum order equal to 2 in the

spacetime derivatives of the fields at each order in the coupling constant. Here, we present the main results without insisting on the cohomology tools required by the technique of consistent deformations. The cohomological proofs are similar to those from [33, 34] and will not be detailed in the following. There appear two distinct solutions to (3.2), which cannot coexist. This is due to the higher-order consistency equations of the deformation procedure. More precisely, both types of solutions survive at the level of S_1, S_2 and S_3, but the existence of S_4 as a solution to the equation $\frac{1}{2}(S_2, S_2) + (S_1, S_3) + (S_4, S) = 0$ is equivalent to the result that they are mutually exclusive (for more details, see appendix B in [34]).

The first type of deformed solution to the master equation (3.2), that is consistent to all orders in the coupling constant, stops at order 1 in the coupling constant and reads

$$\bar{S} = S + \frac{g}{3 \cdot 4!} \int \mathrm{d}^5 x \, \varepsilon^{\lambda\mu\nu\rho\kappa} F_{\lambda\mu} F_{\nu\rho} V_{\kappa}, \tag{4.1}$$

where S is given by (2.19) with $D = 5$. It is important to stress that this result is obstructed to higher dimensions, being the *only* possibility in $D \geqslant 5$ that complies with *all* of our working hypotheses. Indeed, the Chern–Simons actions in $D > 5$, $\int \mathrm{d}^{2k+1} x \, \varepsilon^{\mu_1\mu_2\dots\mu_{2k-1}\mu_{2k}\mu_{2k+1}} F_{\mu_1\mu_2} \cdots F_{\mu_{2k-1}\mu_{2k}} V_{\mu_{2k+1}}$, with $k > 2$, are ruled out by the derivative-order assumption since they contain $k > 2$ spacetime derivatives. The case described by (4.1) is not interesting since it provides no cross-couplings between the vector field and the tensor field with the mixed symmetry $(3, 1)$. It simply restricts the free Lagrangian action (2.1) to evolve on a five-dimensional spacetime and adds to it a generalized Abelian Chern–Simons term, without changing the original gauge transformations (2.4) and (2.5) and, in consequence, neither the original Abelian gauge algebra nor the reducibility structure.

The second type of full deformed solution to the master equation (3.2) ends at order 2 in the coupling constant and is given by

$$\bar{S} = S + g \sum_{A=1}^{n} \left[y^A \int \mathrm{d}^5 x \, \varepsilon^{\lambda\mu\nu\rho\kappa} \left(V_{\lambda}^* \mathcal{F}_{\mu\nu\rho\kappa}^A - \frac{2}{3} F_{\lambda\mu} \partial_{[\xi} t_{\nu\rho\kappa]|\theta}^A \sigma^{\theta\xi} \right) \right]$$
$$+ \frac{16 g^2}{3} \sum_{A,B=1}^{n} \left[y^A y^B \int \mathrm{d}^5 x \left(\partial_{[\xi} t_{\nu\rho\kappa]|\theta}^A \sigma^{\theta\xi} \right) \partial^{[\xi'} t^{B\nu\rho\kappa]|\theta'} \sigma_{\theta'\xi'} \right], \tag{4.2}$$

where all $\mathcal{F}_{\mu\nu\rho\kappa}^A$ have the pure ghost number equal to 1 and are defined as the antisymmetrized first-order derivatives of the ghosts $\eta_{\nu\rho\kappa}^A$ from the sector $(3, 1)$

$$\mathcal{F}_{\mu\nu\rho\kappa}^A \equiv \partial_{[\mu} \eta_{\nu\rho\kappa]}^A. \tag{4.3}$$

These are in fact the only nontrivial elements with the pure ghost number equal to 1 from the cohomology of the exterior derivative along the gauge orbits, $H(\gamma)$. The quantities y^A are n arbitrary, real numbers and $\varepsilon^{\lambda\mu\nu\rho\kappa}$ is the Levi-Civita symbol in $D = 5$. We observe that this solution 'lives' also in a five-dimensional spacetime, just like the previous one. Of course, there appears the natural question whether (4.2) can be generalized to higher dimensions. The answer is again negative (like with respect to (4.1)), but for quite different reasons. Without entering too many details, we will expose here only the main argument for the existence of these obstructions. If one analyzes separately the first-order deformation of the solution to the master equation in the cross-interacting sector[1], then it can be shown (see appendix A of [34], section 2) that S_1 ends nontrivially at the antighost number 1

$$S_1 = \int \mathrm{d}^D x (a_0 + a_1), \qquad \mathrm{agh}(a_i) = i, \qquad i = 0, 1, \tag{4.4}$$

---

[1] Meaning that we search only solutions S_1 to equation (3.4) that effectively couple BRST generators from the vector sector with those belonging to the mixed-symmetry sector.

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with a_i solutions to the equations

$$\gamma a_1 = 0, \qquad \delta a_1 + \gamma a_0 = \partial_\mu j_0^\mu, \qquad \mathrm{agh}\left(j_0^\mu\right) = 0. \tag{4.5}$$

Looking at (2.8), it is easy to see that the general form of the (nontrivial) solution to the former equation from (4.5) reads

$$a_1 = \sum_{A=1}^{n} \left(t^{*A\lambda\mu\nu|\alpha} M_{\lambda\mu\nu\alpha}^A \eta + V_\lambda^* N^{A\lambda\mu\nu\rho\kappa} \mathcal{F}_{\mu\nu\rho\kappa}^A\right), \tag{4.6}$$

where $M_{\lambda\mu\nu\alpha}^A$ and $N^{A\lambda\mu\nu\rho\kappa}$ are γ-closed quantities built out of the original fields (which is the same with gauge-invariant elements since γ acts on the original fields through the gauge transformations modulo replacing the gauge parameters with the ghosts) in order to ensure $\gamma a_1 = 0$. Since the most general gauge-invariant quantities of the free model are the Abelian field strength $F_{\mu\nu}$, the 'curvature' tensors $K_{\lambda\mu\nu\rho|\alpha\beta}^A \equiv \partial_\alpha \partial_{[\lambda} t_{\mu\nu\rho]|\beta}^A - \partial_\beta \partial_{[\lambda} t_{\mu\nu\rho]|\alpha}^A$ and their derivatives, it follows that the tensors M^A and N^A appearing in (4.6) are polynomials in F, K^A, and their subsequent derivatives (up to a finite order in order to render local deformations). Imposing the derivative-order assumption, it follows immediately that the functions of type M^A are restricted to be at most linear in F, while all N^A must be constant (since otherwise one infers interaction vertices with more than two spacetime derivatives). Requiring the Lorentz covariance and Poincaré invariance, it follows that the only possible candidates are

$$M_{\lambda\mu\nu\alpha}^A = w^A F_{\lambda\mu} \sigma_{\nu\alpha}, \qquad N^{A\lambda\mu\nu\rho\kappa} = y^A \delta_5^D \varepsilon^{\lambda\mu\nu\rho\kappa}, \tag{4.7}$$

with w^A and y^A some arbitrary, real constants and δ_5^D the Kronecker symbol. Inserting (4.7) into (4.6) and acting with δ on the resulting expression, it can be shown (see appendix A of [34], section 2) that the latter equation in (4.5) does not possess solutions with respect to a_0 unless

$$w^A = 0, \qquad A = \overline{1, n}. \tag{4.8}$$

Inserting (4.8) into (4.7) and the corresponding functions into (4.6), we find that the last component of S_1 takes the general form

$$a_1 = \delta_5^D \varepsilon^{\lambda\mu\nu\rho\kappa} V_\lambda^* \left[\sum_{A=1}^{n} \left(y^A \mathcal{F}_{\mu\nu\rho\kappa}^A\right)\right], \tag{4.9}$$

which is nothing but the first term from the sum in the right-hand side of (4.2) for $D = 5$. Starting with this only possibility for a_1, it is merely a matter of computation to show that the corresponding deformed solution to the master equation, which is consistent to *all* orders in the coupling constant, is precisely (4.2). We can thus state that the source of obstructions to generalizations of (4.2) in higher dimensions ($D > 5$) is complex, being given by a combination of *all* hypotheses: locality, Lorentz covariance, Poincaré invariance and derivative-order assumption.

From (4.2) we read all the information on the gauge structure of the coupled theory. The terms of the antighost number zero in (4.2) provide the Lagrangian action. They can be equivalently organized as

$$\bar{S}_0\left[t_{\lambda\mu\nu|\alpha}^A, V_\mu\right] = S_0^{\mathrm{t}}\left[t_{\lambda\mu\nu|\alpha}^A\right] - \frac{1}{4} \int \mathrm{d}^5 x \, \bar{F}_{\mu\nu} \bar{F}^{\mu\nu}, \tag{4.10}$$

in terms of the deformed field strength

$$\bar{F}^{\mu\nu} = F^{\mu\nu} + \frac{4g}{3} \varepsilon^{\mu\nu\alpha\beta\gamma} \sum_{A=1}^{n} \left(y^A \partial_{[\rho} t_{\alpha\beta\gamma]}^A{}^{|\rho}\right), \tag{4.11}$$

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where $S_0^t\big[t^A_{\lambda\mu\nu|\alpha}\big]$ is the Lagrangian action of the massless tensor fields $t^A_{\lambda\mu\nu|\alpha}$ appearing in (2.1) with $D=5$. We observe that action (4.10) contains only mixing-component terms of order 1 and 2 in the coupling constant. The piece of antighost number 1 appearing in (4.2) gives the deformed gauge transformations in the form

$$\bar{\delta}_{\epsilon,\chi} t^A_{\lambda\mu\nu|\alpha} = -3\partial_{[\lambda}\epsilon^A_{\mu\nu\alpha]} + 4\partial_{[\lambda}\epsilon^A_{\mu\nu]\alpha} + \partial_{[\lambda}\chi^A_{\mu\nu]|\alpha}, \tag{4.12}$$

$$\bar{\delta}_{\epsilon,\chi} V^\mu = \partial^\mu\epsilon + 4g\varepsilon^{\mu\alpha\beta\gamma\delta}\sum_{A=1}^n\big(y^A\partial_\alpha\epsilon^A_{\beta\gamma\delta}\big). \tag{4.13}$$

It is interesting to note that only the gauge transformations of the vector field are modified during the deformation process. This is enforced at order 1 in the coupling constant by a term linear in the antisymmetrized first-order derivatives of some gauge parameters from the $(3,1)$ sector. At antighost numbers strictly greater than 1, (4.2) coincides with solution (2.19) corresponding to the free theory. Consequently, the gauge algebra and the reducibility structure of the coupled model are not modified during the deformation procedure, being the same as in the case of the starting free action (2.1) subject to the gauge transformations given in (2.4) and (2.5). It is easy to see from (4.10), (4.12) and (4.13) that if we impose the PT-invariance at the level of the coupled model, then we obtain no interactions (we must set $g=0$ in these formulae).

Action (4.10) seems to couple the vector field to each field $t^A_{\lambda\mu\nu|\alpha}$ (assuming that all y^A are nonvanishing) and also to provide cross-couplings between different fields $t^A_{\lambda\mu\nu|\alpha}$ (see the last term from the right-hand side of (4.2) with $A\neq B$). We will show that it is in fact possible to redefine both the fields $t^A_{\lambda\mu\nu|\alpha}$ and the constants y^A such that: (1) the vector field gets coupled to a single mixed-symmetry tensor field from the collection and (2) the cross-couplings between different fields $t^A_{\lambda\mu\nu|\alpha}$ are discarded. In order to show this result, let us denote by Y the matrix of elements $y^A y^B$. It is simple to see that the rank of Y is equal to 1. By an orthogonal transformation M, we can always find a matrix $\hat{Y}$ of the form

$$\hat{Y} = M^T Y M, \tag{4.14}$$

with M^T the transpose of M, such that $\hat{Y}$ is diagonalized and a single diagonal element (for definiteness, we take the first) is nonvanishing

$$\hat{Y}^{11} = \sum_{A=1}^n (y^A)^2 \equiv y^2, \qquad \hat{Y}^{1A'} = \hat{Y}^{B'1} = \hat{Y}^{A'B'} = 0, \qquad A', B' = \overline{2,n}. \tag{4.15}$$

If we make the notation

$$\hat{y}^A = M^{AC} y^C, \tag{4.16}$$

then relation (4.15) implies

$$\hat{y}^A = y\delta_1^A. \tag{4.17}$$

Now, we make the linear field redefinition

$$t^A_{\lambda\mu\nu|\alpha} = M^{AC}\hat{t}^C_{\lambda\mu\nu|\alpha}, \tag{4.18}$$

with M^{AC} the elements of M. It is easy to see that this transformation leaves $S_0^t\big[t^A_{\lambda\mu\nu|\alpha}\big]$ invariant (it remains equal to the sum of free actions, one for every transformed field $\hat{t}^A_{\lambda\mu\nu|\alpha}$ from the collection) and, moreover, the deformed action (4.10) becomes

$$\bar{S}_0\big[t^A_{\lambda\mu\nu|\alpha}, V_\mu\big] = S_0^t\big[\hat{t}^A_{\lambda\mu\nu|\alpha}\big] - \frac{1}{4}\int d^5x\, \bar{F}'_{\mu\nu}\bar{F}'^{\mu\nu}, \tag{4.19}$$

where

$$\bar{F}'^{\mu\nu} = F^{\mu\nu} + \frac{4g}{3} y \varepsilon^{\mu\nu\alpha\beta\gamma} \partial_{[\rho} \hat{t}^1_{\alpha\beta\gamma]|}{}^{\rho}. \tag{4.20}$$

Action (4.19) is invariant under the gauge transformations

$$\delta_{\hat{\epsilon},\hat{\chi}} \hat{t}^A_{\lambda\mu\nu|\alpha} = -3\partial_{[\lambda} \hat{\epsilon}^A_{\mu\nu\alpha]} + 4\partial_{[\lambda} \hat{\epsilon}^A_{\mu\nu]\alpha} + \partial_{[\lambda} \hat{\chi}^A_{\mu\nu]|\alpha}, \tag{4.21}$$

$$\delta_{\hat{\epsilon},\hat{\chi}} V^\mu = \partial_\mu \epsilon + 4gy\varepsilon^{\mu\alpha\beta\gamma\delta} \partial_\alpha \hat{\epsilon}^1_{\beta\gamma\delta}, \tag{4.22}$$

where the new gauge parameters are

$$\hat{\epsilon}^A_{\mu\nu\alpha} = \epsilon^B_{\mu\nu\alpha} M^{BA}, \qquad \hat{\chi}^A_{\mu\nu|\alpha} = \chi^B_{\mu\nu|\alpha} M^{BA}. \tag{4.23}$$

It is now clear that (4.19) decomposes into the action inferred in [34] that couples only the first tensor field with the mixed symmetry $(3, 1)$ from the collection $(A = 1)$ to the vector field and the sum of free actions for the remaining $(n - 1)$ tensor fields with the mixed symmetry $(3, 1)$. In conclusion, one cannot couple different fields with the mixed symmetry $(3, 1)$ through a vector field. A single field of this kind may be coupled nontrivially in $D = 5$, while the others remain free.

It is important to stress that the problem of obtaining consistent interactions depends strongly on the spacetime dimension. For instance, if one starts with action (2.1) in $D > 5$, then one inexorably gets $\bar{S} = S$, so *no* term can be added to either the original Lagrangian or its gauge transformations.

5. Generalization to an arbitrary p

Although the main results discussed so far do not admit generalizations to $D > 5$ for a vector field, there exists a possible generalization if one extends the form degree from one to an arbitrary p. In this situation, the starting point is given by a free model describing a collection of n massless tensor fields $t^A_{\lambda\mu\nu|\alpha}$ and an Abelian p-form

$$S_0\big[t^A_{\lambda\mu\nu|\alpha}, V_{\mu_1\ldots\mu_p}\big] = S_0^{\mathrm{t}}\big[t^A_{\lambda\mu\nu|\alpha}\big] + S_0^{\mathrm{V}}[V_{\mu_1\ldots\mu_p}], \tag{5.1}$$

where

$$S_0^{\mathrm{V}}[V_{\mu_1\ldots\mu_p}] = -\frac{1}{2 \cdot (p+1)!} \int \mathrm{d}^D x \, F_{\mu_1\ldots\mu_{p+1}} F^{\mu_1\ldots\mu_{p+1}} \tag{5.2}$$

and $S_0^{\mathrm{t}}\big[t^A_{\lambda\mu\nu|\alpha}\big]$ follows from formula (2.1). The spacetime dimension is subject to the inequality

$$D \geqslant \max(5, p + 1), \tag{5.3}$$

which ensures that the number of physical degrees of freedom of this free model is non-negative. The Abelian p-form field strength is defined in the usual manner as

$$F_{\mu_1\ldots\mu_{p+1}} = \partial_{[\mu_1} V_{\mu_2\ldots\mu_{p+1}]}. \tag{5.4}$$

Action (5.1) is invariant under a generating set of gauge transformations given by (2.4) for the fields $t^A_{\lambda\mu\nu|\alpha}$ and by

$$\delta_{\overset{(1)}{\rho}} V_{\mu_1\ldots\mu_p} = \partial_{[\mu_1} \overset{(1)}{\rho}{}_{\mu_2\ldots\mu_p]} \tag{5.5}$$

for the Abelian p-form, where the gauge parameters $\overset{(1)}{\rho}{}_{\mu_1\ldots\mu_{p-1}}$ are completely antisymmetric. The gauge symmetries of $S_0^{\mathrm{t}}\big[t^A_{\lambda\mu\nu|\alpha}\big]$ are reducible of order 2, while the gauge transformations

(5.5) are reducible of order $(p - 1)$, such that the overall reducibility order will be equal to $\max(2, p - 1)$.

The BRST algebra contains two types of generators: some from the collection sector, described previously, and the others from the p-form sector. The latter generators comprise the field $V_{\mu_1...\mu_p}$ and its antifield $V^*_{\mu_1...\mu_p}$, the ghosts $(\overset{(k)}{\xi}_{\mu_1...\mu_{p-k}})_{k=\overline{1,p}}$ corresponding to the gauge parameters $(k = 1)$ and to the reducibility functions $(k = \overline{2, p})$, together with their antifields $(\overset{(k)*}{\xi}_{\mu_1...\mu_{p-k}})_{k=\overline{1,p}}$ (all these generators define, where appropriate, antisymmetric tensors). The solution to the master equation for this free model takes the simple form

$$S = S^{\mathrm{t}} + S^{\mathrm{V}}, \tag{5.6}$$

where S^{t} follows from (2.19) and S^{V} is expressed by

$$S^{\mathrm{V}} = S_0^{\mathrm{V}}[V_{\mu_1...\mu_p}] + \int \mathrm{d}^D x \left(V^{*\mu_1...\mu_p} \partial_{[\mu_1} \overset{(1)}{\xi}_{\mu_2...\mu_p]} + \sum_{k=1}^{p-1} \overset{(k)*\mu_1...\mu_{p-k}}{\xi} \partial_{[\mu_1} \overset{(k+1)}{\xi}_{\mu_2...\mu_{p-k}]} \right). \tag{5.7}$$

Although the cohomological structure in the case of a p-form with $p > 1$ is clearly richer than in the presence of a vector field, nevertheless the cohomology of the tensor fields with the mixed symmetry (3, 1) is dominant. Just like in the previous situation of a vector field, there appear two types of fully deformed solutions to the master equation, *which again cannot coexist*. We cannot stress enough that these results take place for the same working hypotheses as in the case of a one-form. The first type generalizes (4.1) and is expressed by

$$\bar{S} = S + g c_1 \delta_{2p+1}^D \int \mathrm{d}^D x \, \varepsilon^{\lambda_1...\lambda_p \mu_1...\mu_{p+1}} V_{\lambda_1...\lambda_p} F_{\mu_1...\mu_{p+1}}$$
$$+ g c_2 \delta_{3p+2}^D \int \mathrm{d}^D x \, \varepsilon^{\lambda_1...\lambda_p \mu_1...\mu_{p+1} \nu_1...\nu_{p+1}} V_{\lambda_1...\lambda_p} F_{\mu_1...\mu_{p+1}} F_{\nu_1...\nu_{p+1}}, \tag{5.8}$$

where S reads as in (5.6), $c_{1,2}$ are two arbitrary, real constants and δ_{2p+1}^D denotes the Kronecker symbol. This situation describes no interactions among the tensor fields $t^A_{\lambda\mu\nu|\alpha}$ or between $t^A_{\lambda\mu\nu|\alpha}$ and the p-form: it simply adds to the original Lagrangian density two Chern–Simons terms (only for p odd, since otherwise they are trivial), without modifying the original gauge symmetries. The only difference from the vector field case is that here two kinds of Chern–Simons terms with at most two spacetime derivatives are admitted (again, for an odd p), while there only one was allowed. This is purely a matter of spacetime dimension since here $2p + 1 > \max(5, p + 1)$ for any odd $p > 1$, while for $p = 1$ we have that $2p + 1 = 3 < \max(5, p + 1) = 5$. The second case is pictured by

$$\bar{S} = S + g \sum_{A=1}^{n} \left[y^A \int \mathrm{d}^{p+4} x \left(\varepsilon^{\lambda_1...\lambda_p \mu\nu\rho\kappa} V^*_{\lambda_1...\lambda_p} \mathcal{F}^A_{\mu\nu\rho\kappa} \right. \right.$$
$$\left. + (-)^p \frac{4}{3 \cdot (p+1)!} \varepsilon^{\lambda_1...\lambda_{p+1}\nu\rho\kappa} F_{\lambda_1...\lambda_{p+1}} \partial_{[\xi} t^A_{\nu\rho\kappa]|\theta} \sigma^{\theta\xi} \right) \right]$$
$$+ \frac{16 g^2}{3} \sum_{A,B=1}^{n} \left[y^A y^B \int \mathrm{d}^{p+4} x \left(\partial_{[\xi} t^A_{\nu\rho\kappa]|\theta} \sigma^{\theta\xi} \right) \partial^{[\xi'} t^{B \, \nu\rho\kappa]|\theta'} \sigma_{\theta'\xi'} \right] \tag{5.9}$$

and generalizes result (4.2). (It is clear that in the limit $p = 1$ formula (5.9) is nothing but (4.2).) It describes a theory in $D = p + 4$ spacetime dimensions that is valid for *any* value (even or odd) $p > 1$, which couples the tensor fields $t^A_{\mu\nu\lambda|\rho}$ to the p-form. Regarding the

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Lagrangian structure of this coupled model, we mention that the terms of antighost number zero present in (5.9) produce the Lagrangian action

$$\bar{S}_0\big[t^A_{\lambda\mu\nu|\alpha}, V_{\lambda_1\ldots\lambda_p}\big] = S^{\mathrm{t}}_0\big[t^A_{\lambda\mu\nu|\alpha}\big] - \frac{1}{2\cdot(p+1)!}\int \mathrm{d}^{p+4}x\, \bar{F}_{\mu_1\ldots\mu_{p+1}}\bar{F}^{\mu_1\ldots\mu_{p+1}}, \tag{5.10}$$

in terms of the deformed field strength

$$\bar{F}^{\mu_1\ldots\mu_{p+1}} = F^{\mu_1\ldots\mu_{p+1}} + (-)^{p+1}\frac{4g}{3}\varepsilon^{\mu_1\ldots\mu_{p+1}\alpha\beta\gamma}\sum_{A=1}^{n}\big(y^A\partial_{[\rho}t^A_{\alpha\beta\gamma]|}{}^{\rho}\big), \tag{5.11}$$

where $S^{\mathrm{t}}_0\big[t^A_{\lambda\mu\nu|\alpha}\big]$ is the free action for the collection of $(3,1)$ mixed-symmetry type tensor fields evolving on a spacetime of dimension $D = p + 4$. The pieces of antighost number 1 from (5.9) emphasize the deformed gauge transformations (4.12) in $D = p + 4$ and

$$\bar{\delta}_{\epsilon,\chi}V^{\lambda_1\ldots\lambda_p} = \partial^{\mu}\epsilon + 4g\varepsilon^{\lambda_1\ldots\lambda_p\mu\nu\rho\kappa}\sum_{A=1}^{n}\big(y^A\partial_\alpha\epsilon^A_{\beta\gamma\delta}\big), \tag{5.12}$$

such that only the gauge symmetries of the p-form are modified. If in (5.10) and its gauge symmetries we perform the transformations (4.17), (4.18) and (4.23), then the cross-couplings among different tensor fields $t^A_{\lambda\mu\nu|\alpha}$ intermediated by a p-form get decoupled and we are led to the same conclusions as in the case of a vector field: *the p-form interacts with a single tensor field* $\big(\hat{t}^1_{\lambda\mu\nu|\alpha}\big)$, *while the remaining* $(n-1)$ *tensor fields with the mixed symmetry* $(3,1)$ *are left free*.

6. Conclusions

The main conclusion of this paper is the proof of rigidity of the couplings of a collection of tensor fields with the mixed symmetry $(3,1)$ to a vector field and actually to an arbitrary p-form gauge field. This means that under some natural assumptions (analyticity of the deformations in the coupling constant, locality, Lorentz covariance, Poincaré invariance and preservation of the number of derivatives on each field), a single mixed-symmetry tensor field from the collection gets coupled to the vector field (or to a p-form). Our final result resembles the well-known fact from general relativity according to which there is one graviton in a given world. This is not a surprise since the action of a free tensor field with the mixed symmetry $(3,1)$ is dual to the linearized gravity (in $D = 6$).

Acknowledgments

References

[1] Curtright T 1985 *Phys. Lett.* B **165** 304
[2] Curtright T and Freund P G O 1980 *Nucl. Phys.* B **172** 413
[3] Aulakh C S, Koh I G and Ouvry S 1986 *Phys. Lett.* B **173** 284
[4] Labastida J M and Morris T R 1986 *Phys. Lett.* B **180** 101
[5] Labastida J M 1989 *Nucl. Phys.* B **322** 185
[6] Burdik C, Pashnev A and Tsulaia M 2001 *Mod. Phys. Lett.* A **16** 731
[7] Zinoviev Yu M 2002 On massive mixed symmetry tensor fields in Minkowski space and (A)dS *Preprint* hep-th/0211233

 [8] Hull C M 2001 *J. High Energy Phys.* JHEP09(2001)027
 [9] Bekaert X and Boulanger N 2004 *Commun. Math. Phys.* **245** 27
[10] Bekaert X and Boulanger N 2003 *Class. Quantum Grav.* **20** S417
[11] Bekaert X and Boulanger N 2003 *Phys. Lett.* B **561** 183
[12] Casini H, Montemayor R and Urrutia L F 2003 *Phys. Rev.* D **68** 065011
[13] Boulanger N, Cnockaert S and Henneaux M 2003 *J. High Energy Phys.* JHEP06(2003)060
[14] de Medeiros P and Hull C 2003 *Commun. Math. Phys.* **235** 255
[15] Bekaert X, Boulanger N and Henneaux M 2003 *Phys. Rev.* D **67** 044010
[16] Zinoviev Yu M 2003 First order formalism for mixed symmetry tensor fields *Preprint* hep-th/0304067
[17] Zinoviev Yu M 2003 First order formalism for massive mixed symmetry tensor fields in Minkowski and (A)dS
        spaces *Preprint* hep-th/0306292
[18] Boulanger N and Gualtieri L 2001 *Class. Quantum Grav.* **18** 1485
[19] Anco S C 2003 *Phys. Rev.* D **67** 124007
[20] Bengtsson A K, Bengtsson I and Brink L 1983 *Nucl. Phys.* B **227** 41
[21] Vasiliev M A 2001 *Nucl. Phys.* B **616** 106
        Vasiliev M A 2003 *Nucl. Phys.* B **652** 407
[22] Sezgin E and Sundell P 2002 *Nucl. Phys.* B **634** 120
[23] Francia D and Sagnotti A 2002 *Phys. Lett.* B **543** 303
[24] Bekaert X, Boulanger N and Cnockaert S 2005 *J. Math. Phys.* **46** 012303
[25] Boulanger N and Cnockaert S 2004 *J. High Energy Phys.* JHEP03(2004)031
[26] Ciobîrcă C C, Cioroianu E M and Saliu S O 2004 *Int. J. Mod. Phys.* A **19** 4579
[27] Boulanger N, Leclercq S and Cnockaert S 2006 *Phys. Rev.* D **73** 065019
[28] Bekaert X, Boulanger N and Cnockaert S 2006 *J. High Energy Phys.* JHEP01(2006)052
[29] Barnich G and Henneaux M 1993 *Phys. Lett.* B **311** 123
[30] Hull C M 2000 *Nucl. Phys.* B **583** 237
[31] Hull C M 2000 *J. High Energy Phys.* JHEP12(2000)007
[32] Casini H, Montemayor R and Urrutia L F 2001 *Phys. Lett.* B **507** 336
[33] Bizdadea C, Ciobîrcă C C, Cioroianu E M, Negru I, Saliu S O and Săraru S C 2003 *J. High Energy Phys.*
        JHEP10(2003)019
[34] Bizdadea C, Ciobîrcă C C, Negru I and Saliu S O 2006 *Phys. Rev.* D **74** 045031